Probability: How Much Can I Earn?

Brief Overview:

The following lessons will provide time for your students to investigate how relative size of an outcome affects its probability. Your students take time to construct and evaluate line plots, line graphs while they ponder the idea of earning an allowance.

NCTM Content Standard/National Science Education Standard**:**

- **Develop and evaluate inferences and predictions that are based on data**
- **Understand and apply basic concepts of probability**
- **Select and use appropriate statistical methods to analyze data**

Grade/Level:

Grades 4-6

Duration/Length:

3 to 4 60-minute sessions

Student Outcomes:

Students will:

- **collect data using observations, surveys, and experiments**
- **relate the correlation between the probability of an outcome occurring in relationship to the size of the event**
- **represent data using tables and graphs such as line plots, bar graphs, and line graphs**
- **Compare different representations of the same data and evaluate how well each representation shows important aspects of the data.**
- **Propose and justify conclusions and predictions that are based on data and design studies to further investigate the conclusions or predictions.**
- **predict the probability of outcomes of simple experiments and test the predictions**

Materials and Resources:

Lesson 1

- **Student Resource 1a & 1b- Response Sheet, 1 per student**
- **Student Resource 2- Spinner A, 6 spinners per page, each student needs one spinner**
- **Teacher Resource 1- Value Choice Sheet (cut apart, fold in half, and place in a container)-1 copy**
- **Hat, bucket, cup, or other container to hold value choices**
- **Teacher Resource Sheet 2- Sample Line Plot - 1 copy**
- **<u>Alexander, Who Used to Be Rich Last Sunday</u> by Judith Viorst**
- **Paper Clips - 1 per student**
- **Teacher Resource 4-#1 Student Observation Sheet- 1 copy**
- **Bingo Chips for re-teaching - 15 per student**

Lesson 2

- **Student work from day 1**
- **Chart Paper- 1 sheet per group**
- **Student Resource 3a & 3b- Samples of line graphs- 1 copy of all four per group**
- **Student Resource 4a or 4b- Venn diagram 1 copy per group -1 transparency**
- **Class Set of Transparencies with Student Resource 6a - Graph line graph copied on them.**
- **Teacher Resource 5-#2 Student Observation Sheet- 1 copy**
- **Paper Clips - 1 per student**

Lesson 3

- **Student Resource 5- Spinner B – one spinner for each pair of students and 1 copy on a transparency for the overhead projector.**
- **Student Resource 6b-Graph Line Plot-1 per student**
- **Student Resource 7-Centemeter Grid Paper one sheet for each student and 1 copy on a transparency for the overhead projector**
- **Student Resource 8-Spinner C and recording sheet- 1 per student**
- **Paper Clips**

- **Student Resource 9-Summative Assessment – 1 per student**
- **Teacher Resource 5a & 5b- Summative Assessment Scoring Guide for teacher use**

Development/Procedures:

Lesson 1
Pre-Assessment

- **Access student knowledge about how people earn an allowance. Talk about factors that could change the amount of money you might earn. Talk about the fact that one person might get $.50 for making a bed, while another person might get $.75. Ask students how they could show how much allowance they have made in a month? (graphs)**

Launch

- **Read <u>Alexander, Who Used to Be Rich Last Sunday</u> by <u>Judith Viorst (Author), Ray Cruz</u> or another book that illustrates earning money. Reintroduce a discussion on earning an allowance.**
- **Use** Teacher Resource 1- Value Choice Sheet **(cut apart and place in a container)**
- **Place folded slips of paper with different amounts in a container for the children to pull randomly.**
- **Students will pick an amount from the container and this will represent the allowance earned daily.**
- **Ask: How much money will you earn by the end of the week? Provide time for the students to think about how to solve this and calculate the total.**
- **Ask them to share the strategy they used to solve this problem. Highlight effective strategies they used to calculate a list of numbers**
- **Ask the students to think about the information they have. You want to know how much money everyone will earn by the end of the week. How might you show this data so it is easily analyzed? Guide the discussion toward making a line plot.**
- **Model how to make a line plot, plotting the data the students calculated. Be sure to include all parts of a well-constructed line plot. See** Teacher Resource 2- Sample Line Plot **as a reference.**
- **Ask the students: What would happen if you earned a different amount each day. How will this change the way you determine the total for the week? Provide independent think time, and then have the students share their ideas with a partner. Finally, allow the class**

to share some ideas with the whole group. The process of adding each day for the entire 7 days should be covered.

Student Application

- **Divide the class in two groups. Half will be given** Student Resource 2-Spinner A **and the other half will use one number cube. Do not allow the groups to see the tool used in the other group. Have the students sit in two different areas of the room so the information is kept top secret. Each value stands for the number of quarters earned each day. 1=.25 2=.50 3=.75 4=1.00 5=1.25 6=1.50. Students will spin the spinner or roll the dice seven times. How much money will you make in one week?**
- **Introduce** Student Resource 1a- Student Response Sheet **to the class. Model how to use a number cube or a spinner and record results for each group.**
- **Give the students 15 minutes to record their individual data.**
- **Have them complete the line plot on** Student Resource 1-Student Response 1b **to show their individual data.**

Embedded Assessment

- **Be sure to observe students as they build line plots to show their results. Jot down any statistical vocabulary you hear during your observations.**

Re-teaching/Extension **–**

- **For those individuals who did not demonstrate understanding of a line plot use bingo chips to represent each quarter. Roll the number cube or spin the spinner with them. Have them place a disk above the outcome each time they roll or spin. The line plot will be built as they play.**
- **Use play quarters for students who are experiencing difficulty with calculating their totals for the day and/or the week.**
- **For those who have understood the lesson, provide them with choices about how to show their results two ways. They can make a line plot and one other representation such as stem and leaf plots. Then they can discuss the different benefits of each type of graph.**

Lesson 2

Pre-assessment –

- **Separate the spinner and number cube group and choose a student to lead each group in a discussion about the students' individual data results displayed on the line plots. Discuss similarities or differences between individual graphs.**
- **Ask the students to address the following questions as they work with their group. Write these questions on cards or on the board for each group.**
  - **What do you notice about your line plot?**
  - **Tell me one observation you notice about your graph compared to the rest of your group.**
  - **What would happen to the trend of our data if we built one graph with everyone's results?**
  - **How will the graph change if we increase the value of each outcome by 2?**
  - **What fractional portion of the cube or spinner does each possible outcome cover?**
- **Introduce the mathematical vocabulary; cluster and outlier to reinforce how they affect the results. An outlier may skew the data and a cluster is where the majority of the results exist.**
- **Circulate during this time and use** Teacher Resource 3-#1 Student Observation Sheet **to make observations.**

Launch –

- **Pull both groups together. Ask: How will the data results change if all of the individual data is combined? Bring in the vocabulary experimental and theoretical data. Explain that the more data that exists the closer the results are to a theoretical data.**
- **Split the groups again to combine data. Each group will combine individual data onto one line plot using a different color for each student. Then they will evaluate, analyze, and discuss the results of the group's combined data.**

Teacher Facilitation –

- **Group students in 4-5 groups.**

- **Distribute one** Student Resource 3a & 3b-Line Graph **for each group to analyze and interpret the trend of the data on all 4 line graphs. The groups will complete** Student Resource 4a or 4b-Venn Diagram **together to point out similarities and differences of the two graphs. Be sure all students' handwriting appears on the Venn diagram. Tell them to pay close attention to the components and purpose of a line graph. Emphasize that a line graph is used to show change over time.**
- **Share the similarities and differences as a class. Record on a transparency or chart paper. Create a list on the board of the components of a good line graph.**

Student Application **–**

- **Provide** Student Resource 6a- Graph **Line Graph sheet of transparency paper for each student. Using a transparency, each student will construct a line graph to show the results of their groups' data. Provide enough time to complete their graphs.**
- **Partner a student from the spinner and number cube group together and have them superimpose the two line graphs.**
- **Independently write silently for two minutes. What conclusions can you draw about the two sets of data? Answers should demonstrate the concept of probability that the spinner gave a better chance to receive a higher allowance and why. Also, how the number cube had an equally likely outcomes where the spinner did not.**
- **Share individual thoughts with a partner.**
- **Adjust and/or add to original ideas.**
- **Partners present to the whole class.**
- **Record summative results as a class and then individually have students jot down conclusions on the class results.**

Embedded Assessment –

- **Monitor the students during their group discussions of line graphs. Especially listen for the use of mathematical vocabulary. Observations will be made during the making of the line graphs by using** Teacher Resource 4 - #2 Student Observation Sheet**. Analyze completed line graphs and the presentation from the partners about the group comparisons. Look for a strong understanding of the probability concept that outcomes are directly affected by the size of the event.**

Re-teaching/Extension **–**

- **For those students who do not demonstrate the understanding of line graphs, model and discuss how to make a line graph. To ensure comprehension relate line graphs to real life. During the silent writing time, either work with these students in a small group or make a Cloze Paragraph for the students to fill in during the writing time.**
- **For those students who understand the concept, have them search through newspapers for examples of line graphs. Have them find a line graph that shows an increase, decrease, or one that shows little change and then in writing compare the three graphs. Have the students create a spinner or number cube that would produce the results on their line graphs.**

Lesson 3 (2-3 days)

Pre-assessment –

- **Tell the students that you have a new number cube to work with. It has 8 sides. (Octahedron) It is numbered 1 to 8.**
- **Draw a line plot in your journals to predict the outcomes you might have after rolling the number cube 24 times.**
- **Walk around and observe the student papers. Be sure that they can explain why they chose to illustrate the outcomes in a certain way. You want to see that they have an understanding of equally likely outcomes.**

Launch –

- **Show** Student Resource 5- Spinner B **on the overhead. Announce that today the stakes are higher. This week you will earn dollars, not cents. What observations can you make about the spinner on the overhead projector? Allow time for the students to write their own ideas for 1 minute. Walk around and observe the students. Ask questions to guide students who are having a difficult time coming up with one idea. Students will save their ideas for later use and discussion.**

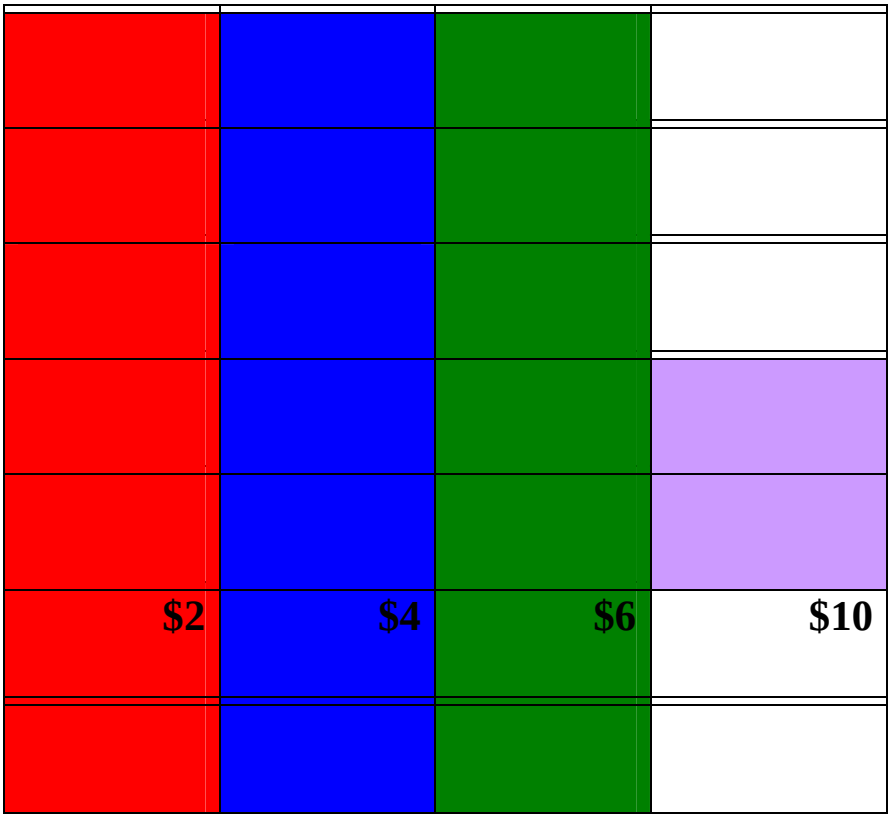
Teacher Facilitation –

- **Model the following exercise. Show students how you work cooperatively with your partner, taking turns spinning the spinner and marking the results. One partner spins 10 times while the second**

person records with tally marks. Then switch jobs. Continue the process until you have a total of 60 trials.

Student Application –

- **Put the students in pairs. Provide each pair of students with** Student Resource 5- Spinner B and Recording Sheet **and a paper clip.**
- **Allow 20 minutes for teams to collect the data.**
- **Analyze the outcomes. Have the students revisit their prediction. Ask: Did your outcomes match what you predicted? Why, or why not?**
- **Provide each student with a sheet of** Student Resource 7-Centimeter Grid Paper**. Make a copy on a transparency for yourself. Model the following process. Choose a color to represent each possible outcome. $2=Red $4=Blue $6=Green $10=Purple. On the grid paper color one square for each tally mark in the corresponding column.**



- **Cut each column. Glue the columns together in one continuous strip. Glue the two ends together to form a circle. Place the circle over the spinner. You do not want to give the secret away, so manipulate your modeled data. Be sure your data does not match the spinner. When the students do this, their circle of cells will usually be a close match to the spinner. Let the students discover this.**
- **Ask what the students think will happen when they do this experiment and make the ring around their spinner.**
- **Have the students create their own ring around their spinner based on their data.**
- **With a partner discuss what you observe. What do you notice? (Note: The area covered by the grid will closely match the area each amount covered on the spinner.) Provide silent writing /thinking time or writing time. Have students record their observations independently. Then share with a partner, and eventually record observations on the board.**

Embedded Assessment –

- **Use** Student Resource 8-Spinner C and Recording Sheet**. Present the following situation.**
  Today you will use the same spinner as yesterday. However, not all will remain the same!!! You will choose one of the possible outcomes to represent money you have to pay back! Be prepared to defend why you are making this decision based on what you have learned about probability over the past few days.
- **Provide** Student Resource 8-Spinner C and Recording Sheet **for all students.**
- **Spin the spinner and record the data. Discuss the results with your partner and analyze the outcomes. Why did the results change? Did you choose a good value to subtract? Why?**
- **Complete the response sheet. Be sure to model strong responses that justify their reasoning. Reference to fractional coverage is one key aspect.**
- **Collect cumulative data from all members in the class to make a comparison of the results. Who earned the most money? What number did they subtract? Use what you learned about probability this week to explain the outcome.**
- **Give the students the Summative assessment to evaluate their understanding of how relative size affects the outcome of an event. Use** Student Resource 9- Summative Assessment. **Allow students the opportunity to discuss their ideas before they write their reasoning on the assessment. Answer key can be found on Teacher Resource.**

Re-teaching/Extension –

- **Re-teach the concept of the greater the area something covers on a spinner, the more likely you are to have that outcome occur.**
- **To extend this activity have the students devise a way to collect data to make a line graph to show the increase and decrease of their allowance throughout the week.**

Summative Assessment**:**

- **Student Resource 9**

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Student Resource 1a

Response Sheet

You have a chance to earn some money this week. How much you earn is not based on how hard you work. The amount you earn is up to chance. Your teacher will give you a spinner or a number cube. The outcome will tell you how many quarters you earn for each day.

Part A

- **Spin the spinner or roll your number cube.**
- **Calculate your total allowance earned for the day.**
- **Record your results on the table below.**

Day	Outcome	Today I earned…	So far I've made…
Sample Day 1	**3**	**.75**	**.75**
Sample Day 2	**6**	**1.50**	**2.25**
1			
2			
3			
4			
5			
6			
7			

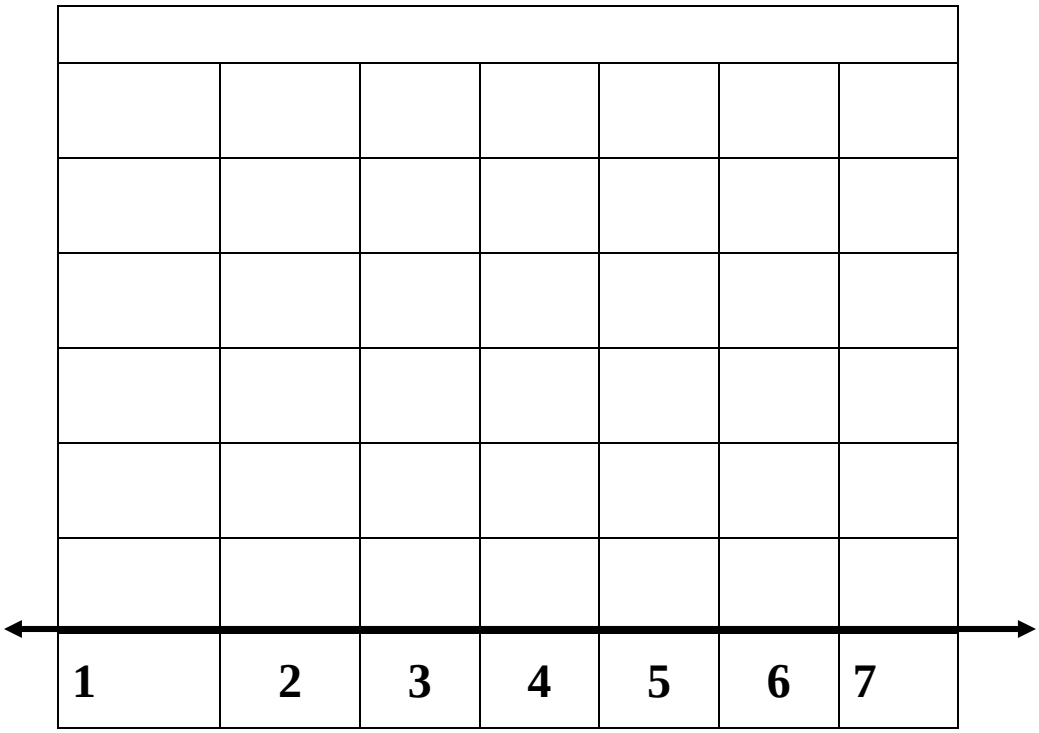
1= 1 quarter	**4=4 quarters**
2=2 quarters	**5=5 quarters**
3=3 quarters	**6=6 quarters**

Student Resource 1b

Response Sheet

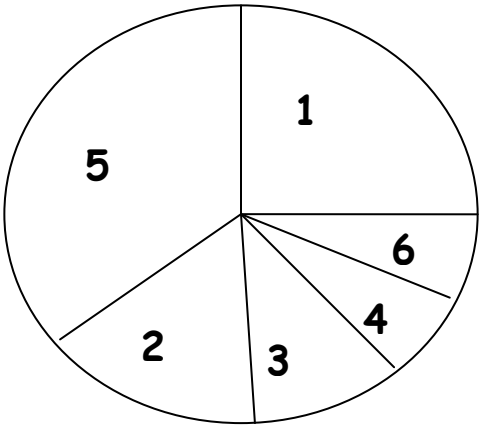
Part 2

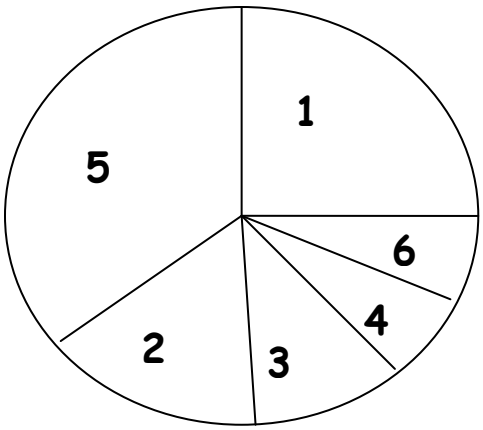
Show off your results. Make a line plot to show how many times you earned each amount.

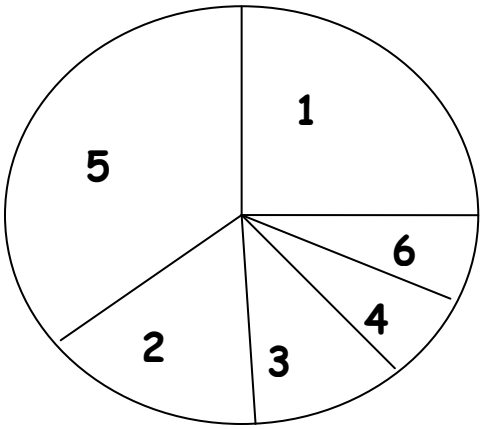


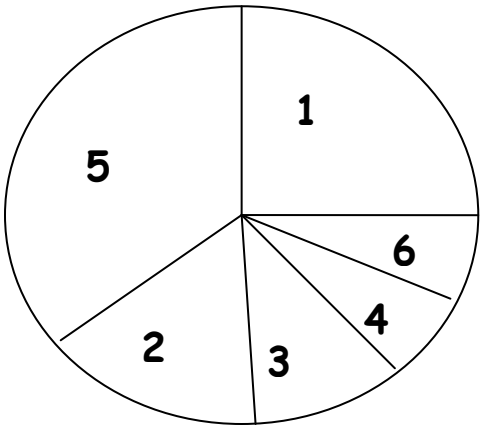
Student Resource 2

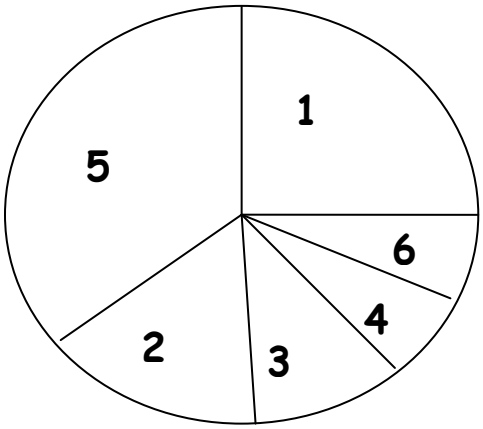
Spinner A

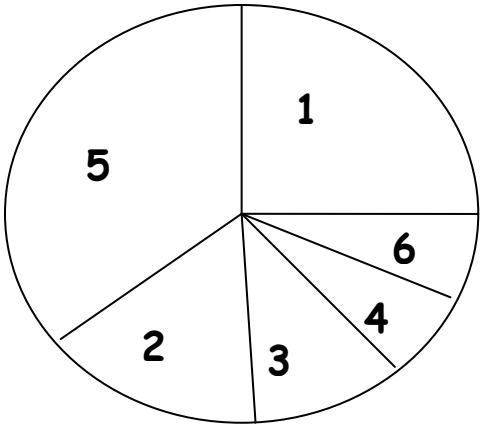






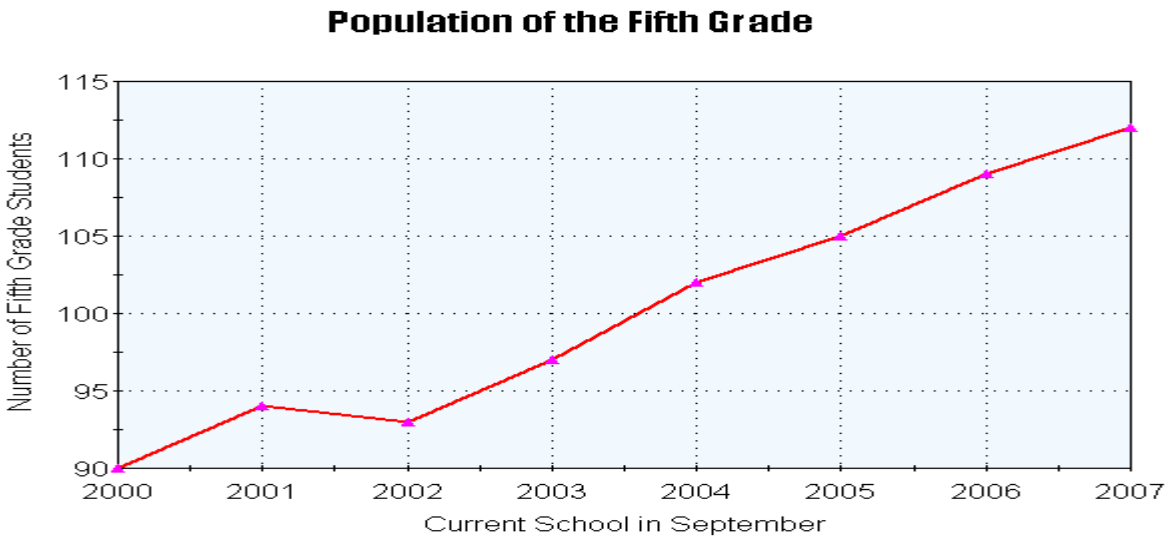


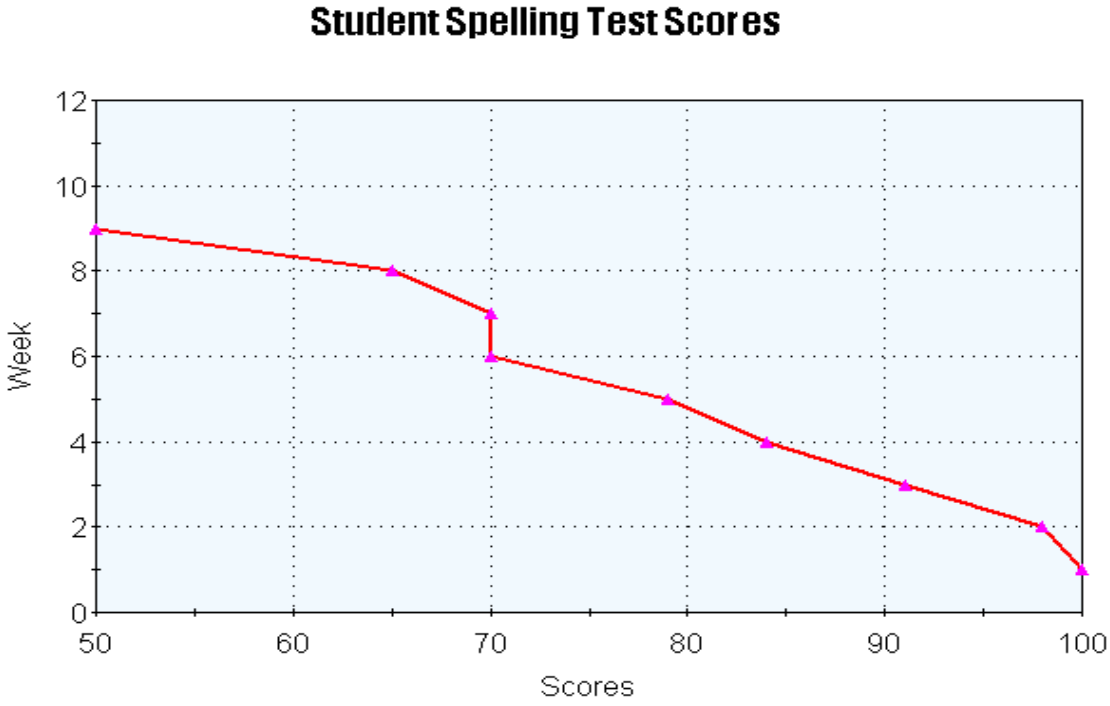




Student Resource 3a

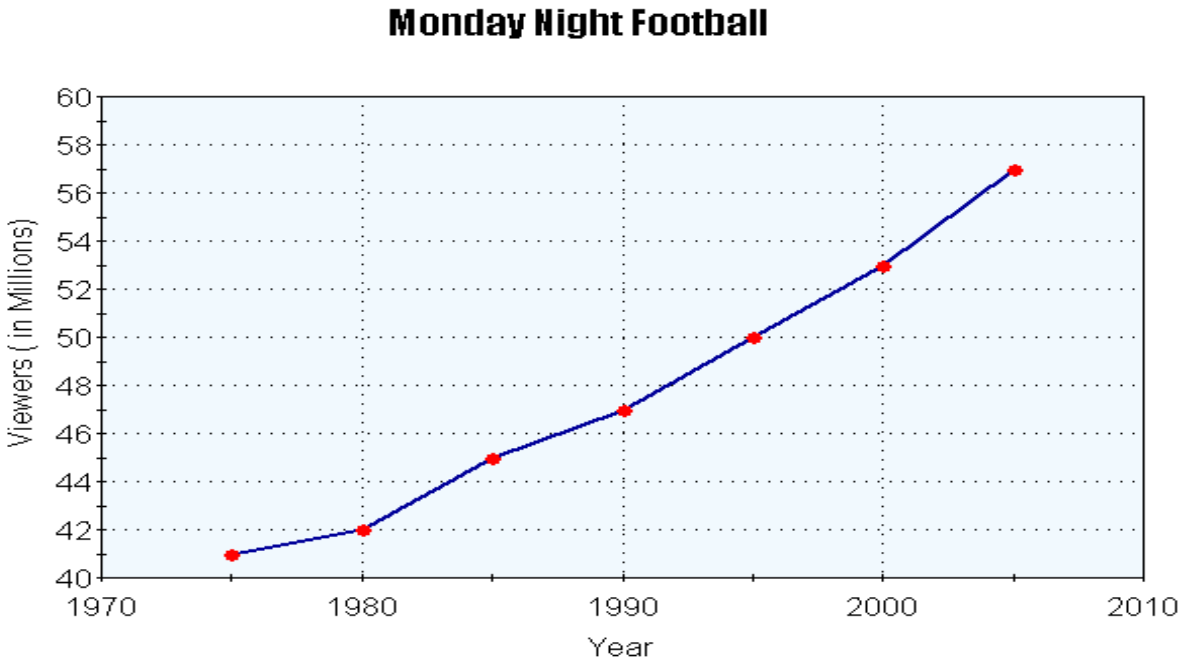
Line Graph Samples

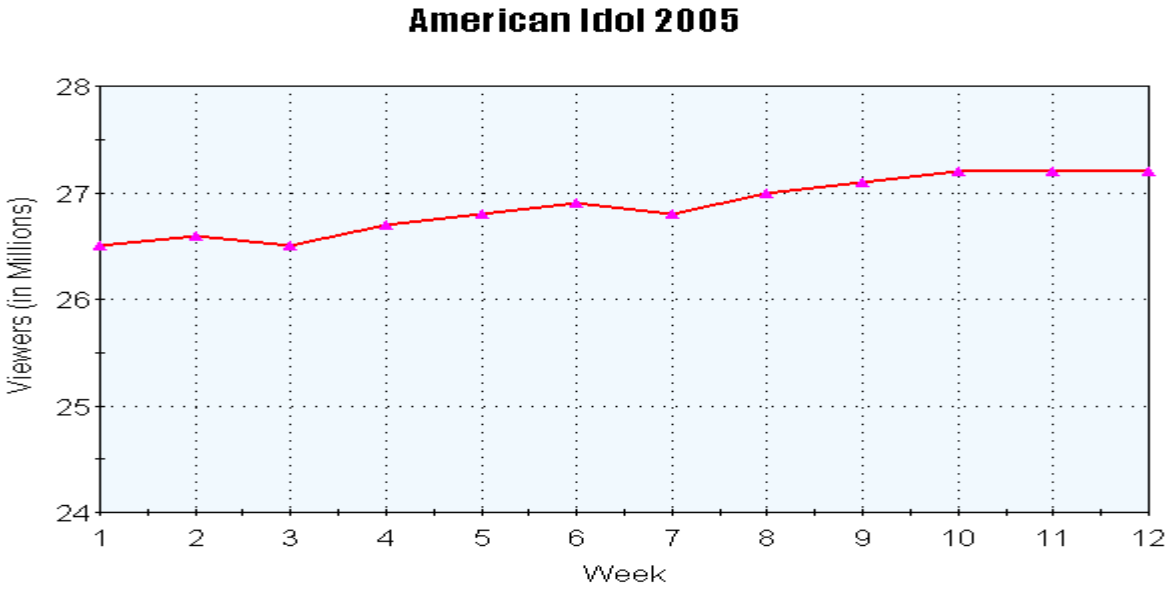




Student Resource 3b

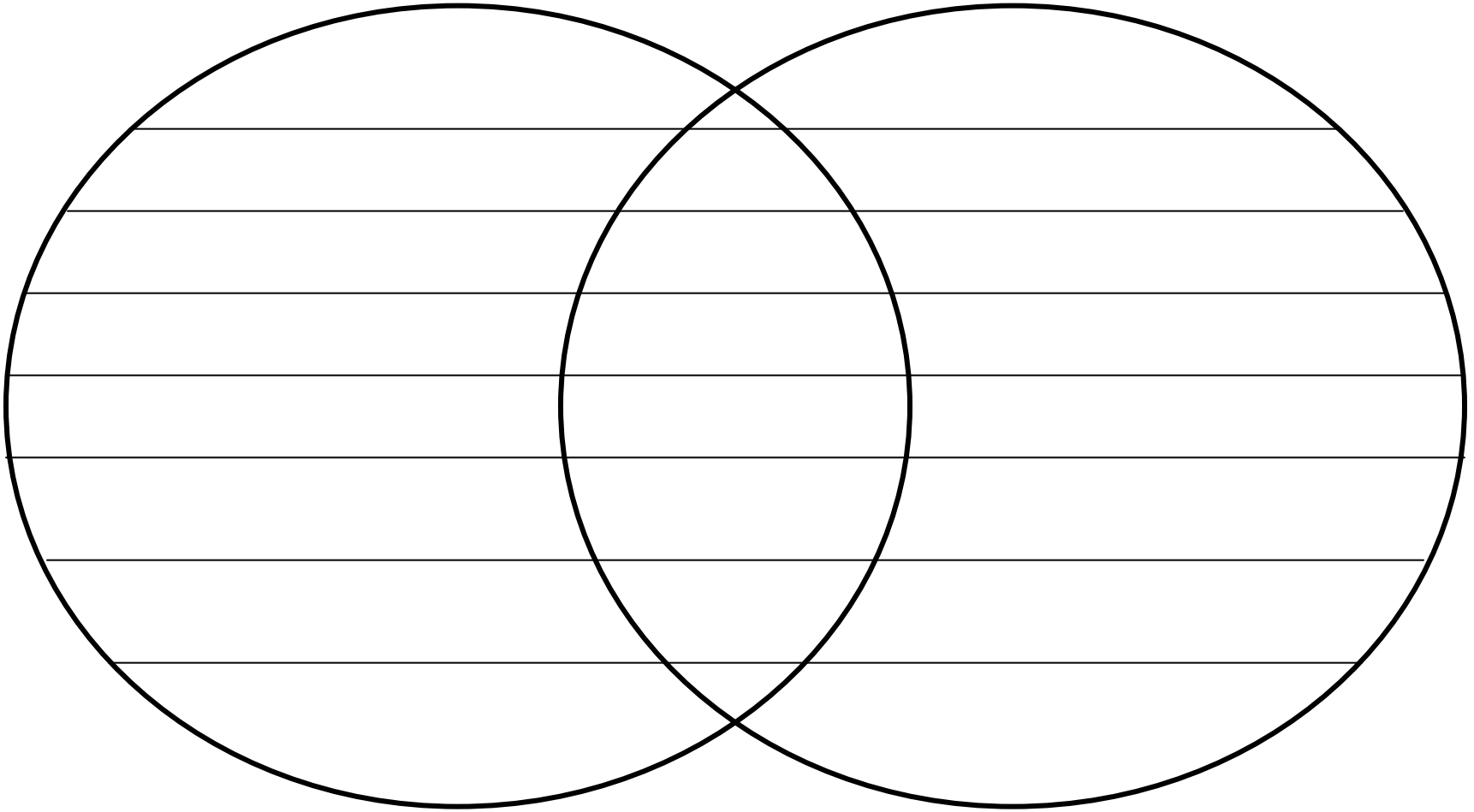
Line Graph Samples





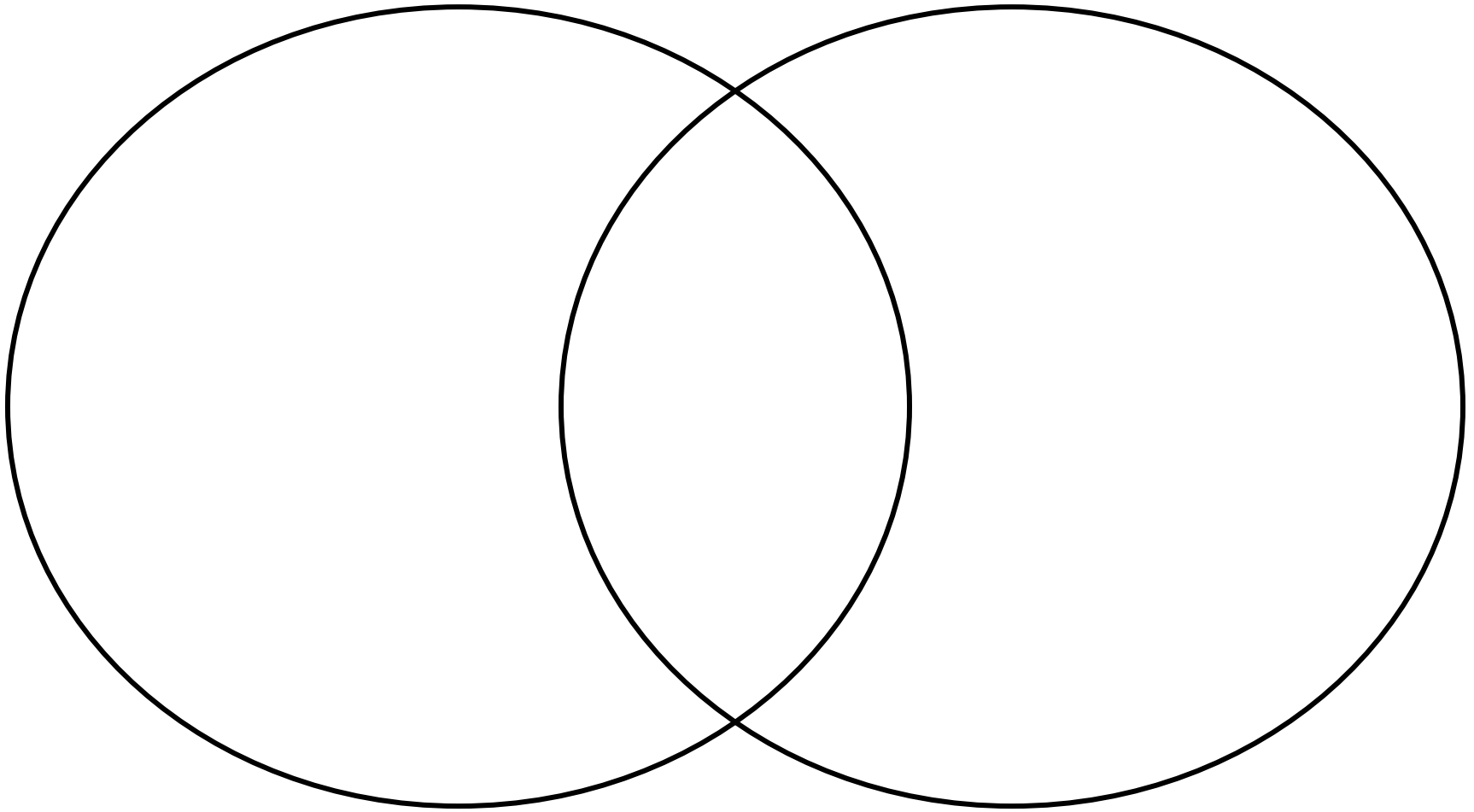
Student Resource 4a

Venn Diagram



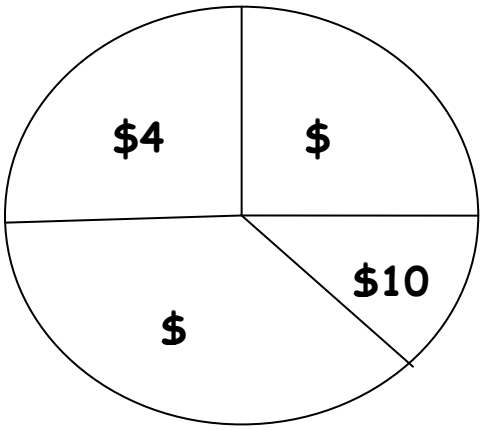
Student Resource 4b

Venn Diagram

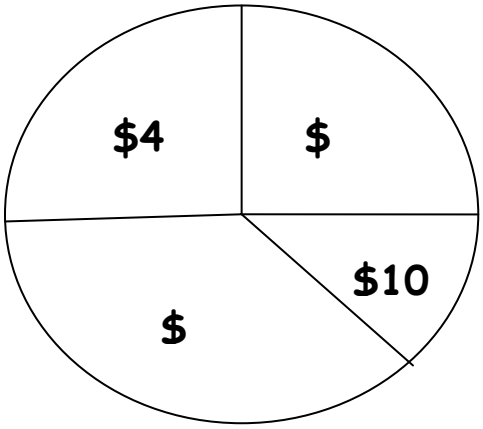


Student Resource 5

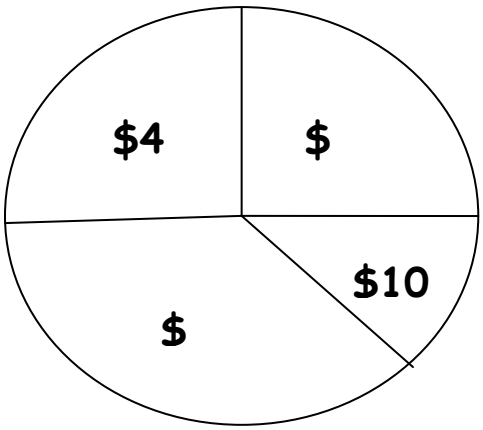
Spinner B and Recording Sheet



Outcome	Frequency
$2	
$4	
$6	
$10	



Outcome	Frequency
$2	
$4	
$6	
$10	

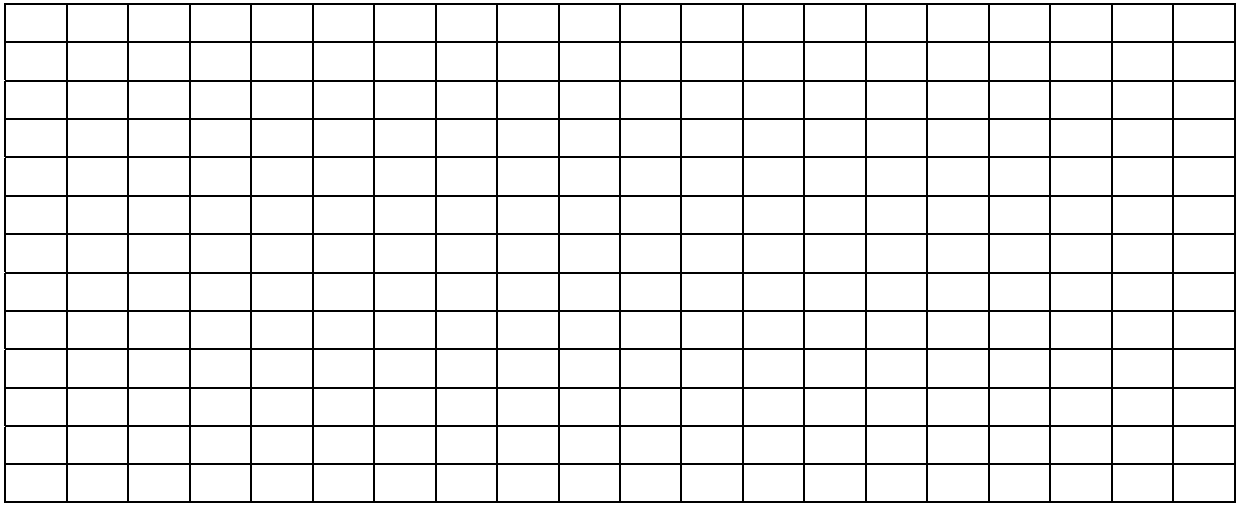


Outcome	Frequency
$2	
$4	
$6	
$10	

Graph

Line Graph

Title



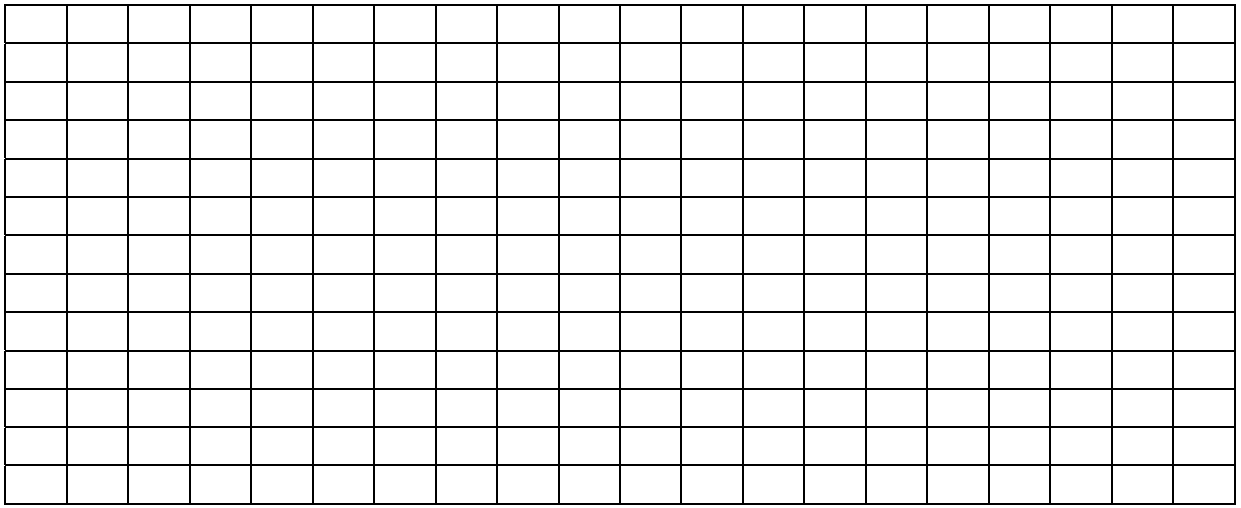
Label

Student Resource 6b

Graph

Line Plot

Title



Label

Student Resource 7

Centimeter Grid Paper

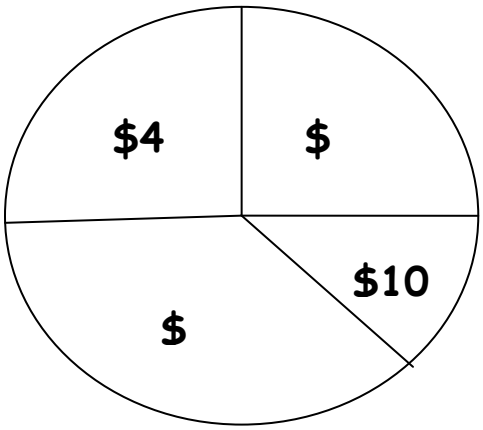
Name _____ Date _____

Student Resource 8

Spinner C & Recording Sheet

Decreased by What?

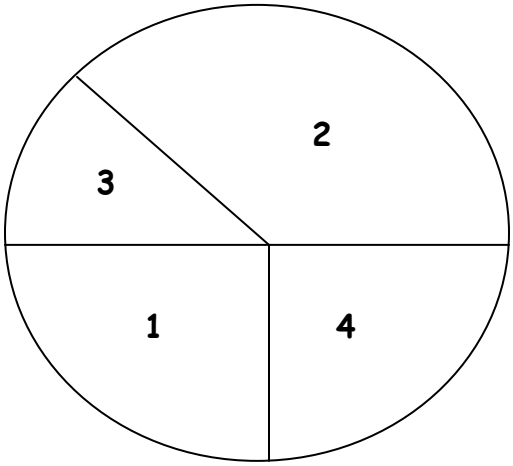
The number I want to subtract is: _____.



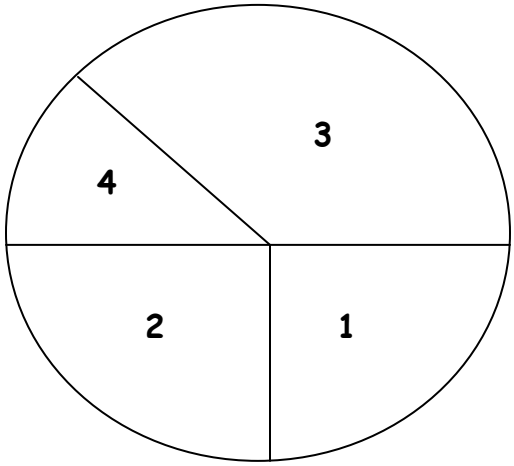
Dollar Amount	Frequency	Total Amount
$2		
$4		
$6		
$10		

My total amount earned is: _____. Use the space below to show your work.

Summative Assessment



Spinner A



Spinner B

Event	Outcome
1	2
2	3
3	3
4	4
5	1
6	3
7	4
8	3
9	2
10	1
11	1
12	3
13	2
14	2
15	1
16	3

Suki's teacher challenged the class. They had to spin a spinner and record the outcome each time. At the end of 16 tries, they had to tally their total score. The person with the highest score got a free homework pass. She was trying to get the largest sum possible. She recorded her results on the table to the left.

Based on the results which spinner did Suki choose to use?

_____

Which spinner would you choose?

You may discuss your thoughts with a partner. Then, on a sheet of lined paper, use what you know about probability to explain why.

Teacher Resource 1

Value Choice Sheet

Directions: Cut the following values apart. Fold each in half. Place all 25 possible outcomes in a container. Each student will pick one value to represent what allowance they will earn each day for one week.

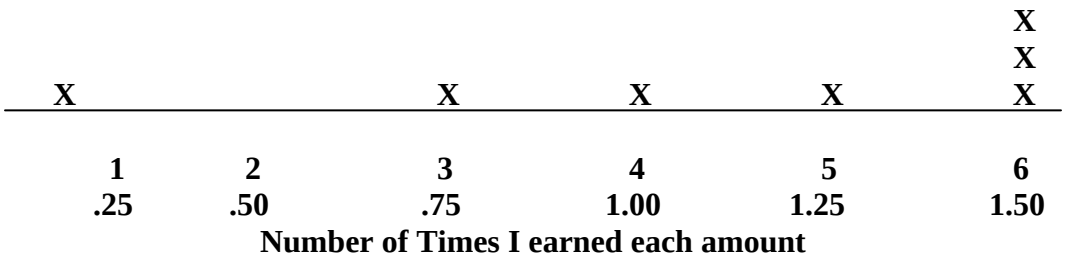
.25	.25	.25
.25	.30	.45
.50	.50	.50
.50	.65	.65
.70	.70	.75
.75	.75	.75
.75	.80	.80
.80	.85	.90

Teacher Resource 2

Sample Line Plot

Day	Outcome	Today I earned…	So far I've made…
Sample Day 1	**3**	**.75**	**.75**
Sample Day 2	**6**	**1.50**	**2.25**
Sample Day 3	**1**	**.25**	**2.50**
Sample Day 4	**6**	**1.50**	**4.00**
Sample Day 5	**4**	**1.00**	**5.00**
Sample Day 6	**6**	**1.50**	**6.50**
Sample Day 7	**5**	**1.25**	**7.75**

How Many Times I Earned Each Amount



Notes

- **2 is included even though there is no data**
- **X's are the same size**
- **There is a title**
- **The line is labeled.**

Teacher Resource 3

#1 Student Observation Sheet

Name	Use of Mathematical Vocabulary	Demonstrate Knowledge of Line Plots

✓+ **Indicates strong understanding**

✓ **Indicates basic understanding**

✓ **Indicates minimal understanding**

Teacher Resource 4

#2 Student Observation Sheet

Name	Use of Mathematical Vocabulary	Demonstrates Knowledge of Line Graphs

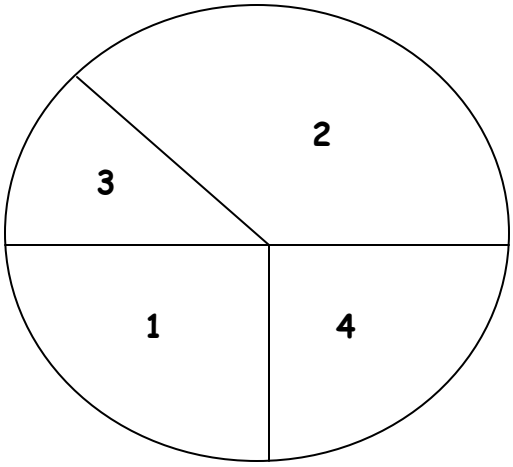
✓+ **Indicates strong understanding**

✓ **Indicates basic understanding**

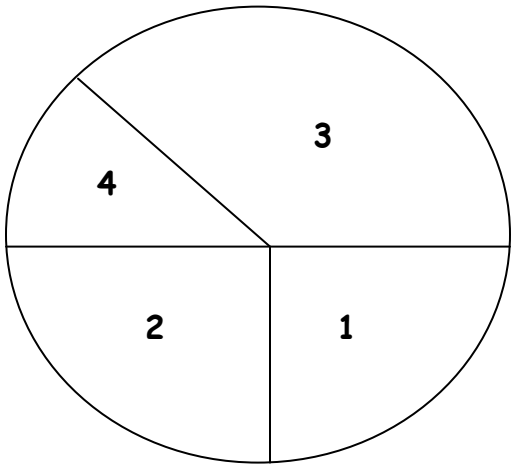
✓ **Indicates minimal understanding**

Teacher Resource 5a

Summative Assessment Scoring Guide



Spinner A



Spinner B

Event	Outcome
1	2
2	3
3	3
4	4
5	1
6	3
7	4
8	3
9	2
10	1
11	1
12	3
13	2
14	2
15	1
16	3

Suki's teacher challenged the class. They had to spin a spinner and record the outcome each time. At the end of 16 tries, they had to tally their total score. The person with the highest score got a free homework pass. She was trying to get the largest sum possible. She recorded her results on the table to the left.

Based on the results which spinner did Suki choose to use?

Spinner B **(1 point)**

Which spinner would you choose?

Either choice could be correct as long as the answer was justified.

You may discuss your thoughts with a partner. Then, on a sheet of lined paper, use what you know about probability to explain why.

Teacher Resource 5b

Summative Assessment Scoring Guide

Scoring:
Justification is clearly stated and the answer demonstrates evidence of understanding the role of probability will play in the outcome.

Sample Justifications (Note: answer clearly supported and justified answer should be considered and accepted.)

Sample A:
After observing the two spinners, I would choose Spinner B. The top half of this spinner gives me a chance to land on the digits 3 or 4, which means that half of Spinner B is worth more than any of the halves of Spinner A. Since my goal is to receive the highest sum, I believe that Spinner B would give me the highest results due to the top half equaling 7.

Sample B:
I would choose Spinner A. When I look at Suki's data, I see she only landed on 4, the highest value 2 times. If I am only going to land on the smallest section of the spinner a few times, I don't want to choose the spinner that doesn't have the highest value in that section. Also, when I compare both spinners, approximately 2/3 of spinner A is 2 and 4, and 2/3 of spinner B is 3 and 1. 2/3 is a much higher value together than 3 and 1. Therefore I think that I will get a higher outcome if I use Spinner A.

Scoring Guide

(3 points)	**The response demonstrates a strong understanding of the probability of an outcome occurring in relationship to the size of the event. Ideas are concretely supported referencing either Suki's data or the fractional representation of outcomes on the spinners.**
(2 Points)	**The response demonstrates a basic understanding of the probability of an outcome occurring in relationship to the size of the event. Ideas are partially supported referencing either Suki's data or the fractional representation of outcomes on the spinners.**
(1 Point)	**The response demonstrates minimal understanding of the probability of an outcome occurring in relationship to the size of the event.**